

Chapter 2
Section 4

Complex Numbers

Imaginary Numbers

- Imaginary Unit: the unit i can be used to write the square root of any negative number.

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$i^3 = -\sqrt{-1} = -i$$

$$i^4 = 1$$

$$i^5 = \sqrt{-1} = i$$

$$i^6 = -1$$

$$i^7 = -\sqrt{-1} = -i$$

$$i^8 = 1$$

Pattern?

Try these

$$1.) i^{27} \quad 27 \div 4 = 6 \text{ R } 3 = i^3 = \boxed{-i}$$

$$2.) i^{54} \quad 54 \div 4 = 13 \text{ R } 2 = i^2 = \boxed{-1}$$

$$3.) i^{65} \quad 65 \div 4 = 16 \text{ R } 1 = i^1 = \boxed{i}$$

$$4.) i^{60} \quad 60 \div 4 = 15 \text{ R } 0 = i^4 = \boxed{1}$$

Complex Numbers

- A complex number written in standard form is a number $a+bi$, where a and b are real numbers.
 - If b does not equal zero, then $a+bi$ is an imaginary number.
 - If $a=0$ and b does not equal zero, then $a+bi$ is a pure imaginary number

Properties of Square Roots of Negatives

- If r is a positive real number, then :

- Example: $\sqrt{-r} = i\sqrt{r}$

- Extra Examples:

$$\sqrt{-5} = i\sqrt{5}$$

Examples:

- 1. Solve

$$x^2 = -4$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = \sqrt{-4}$$

$$x = -2i \text{ and } 2i$$

$$x = \pm 2i$$

Adding and Subtracting Complex Numbers

- Adding:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

- Subtracting:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Examples:

- 1. $(4 - i) + (3 + 2i)$

$$7 + i$$

- 2. $(7 - 5i) - (1 + 5i)$

$$6 - 10i$$

Multiplying and Dividing Complex Numbers

- Multiplying: Use same properties you would when multiplying real numbers
- Dividing: Multiply fraction by complex conjugate (the opposite of what the denominator is)

Examples

$$i^2 = -1$$

• 1. $5i(-2 + i)$

$$-10i + 5i^2$$

$$-10i + 5(-1)$$

$$-10i - 5$$

$$\boxed{-5 - 10i}$$

• 2. $(3 + 4i)(-5 - 2i)$

$$-15 - 6i - 20i - 8i^2$$

$$-15 - 26i - 8(-1)$$

$$-15 - 26i + 8$$

$$\boxed{-7 - 26i}$$

Examples

$$3.) \frac{(5+3i)}{(1-2i)} \cdot \frac{(1+2i)}{(1+2i)}$$

$$\frac{5+10i+3i+\cancel{6i^2}^{-6}}{1+\cancel{2i}-\cancel{2i}-\cancel{4i^2}^{-4}}$$

$$\frac{-1+13i}{5}$$

$$4.) \frac{(4+2i)}{(3+i)} \cdot \frac{(3-i)}{(3-i)}$$

$$\frac{12-4i+6i-\cancel{2i^2}^{-2}}{9-\cancel{3i}+\cancel{3i}-\cancel{i^2}^{-1}}$$

$$\frac{14+2i}{10}$$

$$= \frac{7+i}{5}$$

Homework

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